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B.M.S. COLLEGE FOR WOMEN

BENGALURU -560004

I SEMESTER END EXAMINATION – APRIL - 2024

M.Sc – MATHEMATICS - ALGEBRA-I
(CBCS Scheme – F+R)

Course Code: MM101T **QP Code: 11001**
Duration: 3 Hours **Max. Marks: 70**

Instructions: ***1) All questions carry equal marks.***
2) Answer any five full questions.

1. (a) If N and M are normal subgroups of G, then prove that $\frac{NM}{M} \cong \frac{N}{N \cap M}$.

   (b) Define (i) Symmetric group S_n (ii) Alternating group A_n. Show that $\frac{S_n}{A_n} \cong \{-1, 1\}$.

   (c) Let $G = (Q, +)$ be the additive group of rationals. If $K = Z$, then show that $\frac{G}{K}$ is isomorphic to $\bar{G}$, where $\bar{G} = \{e^{2\pi i\theta} : \theta \in Q\}$ is a group under multiplication.

(6+4+4)

2. (a) Let G be a finite group and S be a finite G -set. If $x \in S$ then show that $O(G_x) = \frac{O(G)}{O(stab\ (x))}$.

   (b) State and prove the Cauchy-Frobenius lemma.

   (c) Verify the class equation for S_3.

(5+5+4)

3. (a) State and prove Sylow's first theorem.

   (b) Show that the number of p-sylow subgroups in G, is $\frac{O(G)}{O(N(P))}$, where P is any p-sylow subgroup of G.

   (c) Show that every group of order 15 is cyclic.

(7+4+3)

4. (a) Show that a normal subgroup N of G is maximal if and only if the quotient group $\frac{G}{N}$ is simple.

   (b) Prove that subgroup of a solvable group is solvable.

   (c) Show that a group of order 28 is solvable but not simple.

(5+5+4)

5. (a) Define a field. Show that every field is an integral domain.

   (b) Let U be a left ideal of a ring R and $\lambda(U) = \{x \in R: xu = 0, \forall\, u \in U\}$. Then show that $\lambda(U)$ is an ideal of the ring R.

   (c) State and prove fundamental theorem of homomorphism of rings.

(3+4+7)

6. (a) Show that an ideal M of the ring of integers is maximal if and only if it is generated by some prime integer in Z.

   (b) Prove that in a principal ideal ring every non-zero prime ideal is maximal.

   (c) Show that the intersection of two prime ideals is also a prime ideal if one is contained in the other.

(6+4+4)

7. (a) Show that the set of Gaussian integers is a Euclidean ring.

   (b) Prove that every Euclidean ring is a principal ideal ring.

   (c) State and prove unique factorization theorem.

(5+5+4)

8. (a) Prove that deg $(fg) = \deg(f) + \deg(g), f, g \in R[x]$. Further if R is an integral domain then show that $R[x]$ is also an integral domain.

   (b) If F is a field, then show that $F[x]$ is not a field.

   (c) Using Eisenstein irreducibility criterion show that $f(x) = x^3 + x^2 - 2x - 1$ is irreducible over Q.

(6+4+4)

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